Emerging Enrollment Trends of Higher Education in USA and India

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Abstract – The aim of this paper is to forecast the educational trend in USA and India. In this context, this paper analyses the USA and Indian education scenario with the help of various mathematical models. Using past data on engineering enrollments in under graduate (U.G.), post graduate (P.G.), and doctorate (Ph.D.) in USA and total enrollments in U.G., P.G., and Ph.D. in India, forecasts have been made through various mathematical models which may be useful for planning and decision making. In this study the use of mathematical modelling-popular in technological forecasting shall also be useful for the purpose of manpower forecasting and policy analysis. The forecasts clearly depict a future scenario, where the enrollments in doctorate are going to be in demand compare to U.G. and P.G. enrollments in USA. On the other hand, students' enrollments in U.G., P.G. and Ph.D. all are continuously increasing in India.

Keywords – Enrollments, Forecasting, Higher Education, India, USA

I. Introduction

THE government is responsible for the health and continued vitality of higher education in the country. The intention is to ensure that a high-quality education is available to every child in the country and that the educational level is sufficient to meet not only national demands but also international requirements. Various problems related with manpower can be solved by mathematical modelling. By using historical data, suitable mathematical models are able to estimate future trends of manpower. Various manpower time series data are useful for relevant mathematical modelling and forecasting. Apart from simple mathematical modelling, in this study the use of mathematical modelling-popular in technological forecasting shall also be used for the purpose of manpower forecasting and policy analysis.

Human resource development (HRD) to meet the national as well as international requirements in new and emerging areas is one of the most significant priorities of our policy makers. Many studies have already been done by Rai and Kumar (2004 and 2006), and Rai and Katiyar (2010) in the past on human resource development in India. To help the educational planners and policy makers assess the current enrollment and doctorate degrees awarded situation (Katiyar et al., 2010).

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An attempt has been made here to analyse and present different scenarios with the help of past data Annual Reports (1995-2006), and Katiyar, R. (2007) by using appropriate mathematical models. The main objective is the evaluation of students' enrollments in engineering with different disciplines in USA and total enrollments in India to get a high level of education and a wide acceptance, keeping in mind many and different requirements, which arise by different needs of different users. The actual data are listed and graphically depicted in the fields of engineering enrollment (USA) in Table I and Fig. 1 respectively. The past data of total enrollment in India are listed in Table VI and graphically depicted in Fig. 3. The data are analysed with the help of mathematical models, which are briefly discussed below.

II. Mathematical Models

A. Linear Model

The linear model equation is:

$$y = a + bx \tag{1}$$

Where 'y' is the dependent variable and 'x' is the independent variable while 'a' and 'b' are the constant coefficients.

B. Exponential Model

The exponential model is defined as:

$$\frac{dy}{dt} = by \tag{2}$$

Integrating equation (2), we get

$$y = ae^{bt} \tag{3}$$

Where 'b' is determined by the physical properties of the substance while 'a' is the initial amount at time zero of the substance.

C. Pearl Model

Forecasting by growth curves requires fitting a mathematical formula for a growth curve to a set of historical data. A very common formula for a growth curve is the Pearl curve, named after the American demographer Raymond Pearl who popularized its use in demographic forecasting (Pearl, 1927). The formula for the Pearl curve is:

$$y = L/(1 + ae^{-bt}) \tag{4}$$

Where 'L' is an upper limit to the growth of the variable represented by 'y', 'a' and 'b' are the coefficients evaluated by "fitting" the curve to the data, and 'e' is the base of the natural logarithms. The curve has an initial value of zero at time $t = -\infty$ and a value of 'L' at time $t = +\infty$. The inflection

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point of this curve occurs at $t = (\log a)/b$, when $y = L/2$. The curve is symmetrical about this point, with the upper half being a reflection of the lower half.

To fit a Pearl curve to a set of data the curve is first 'straightened out'; which requires the following algebraic transformation of the Pearl curve formula.

$$Y = \log(L - y)/y = \log a - bt \quad (5)$$

When $\log(L - y)/y$ is plotted against time, we get a straight line, which can be extrapolated, into the future. The constant term from the regression is '*log a*', and the slope term from the regression is '*b*'. The coefficient '*b*' is to be intrinsically positive, so the line representing the transformed variable '*Y*' slopes down to the right. Once parameters '*a*' and '*b*' are estimated, they can be substituted back into the model to extrapolate the future values.

D. Gompertz Model

This is another growth curve that is frequently used in technological forecasting. This is known as Gompertz curve (Gompertz, 1825) and is represented as

$$y = Le^{-be^{-kt}} \quad (6)$$

The Gompertz curve ranges from zero at $t = -\infty$ to the upper limit '*L*' at $t = +\infty$. Where '*y*' = parameters of technological growth or functional capability; '*L*' = upper limit of the parameter; '*b*' and '*k*' are constants, and '*t*' = time. The curve is not symmetrical and inflection point occurs at

$$[\, t = (\log b)/k,\ y = L/e \,] \quad (7)$$

Just as with the Pearl curve; it is necessary to straighten out the Gompertz curve before linear regression can be used to obtain the coefficients '*b*' and '*k*'. The equation can be rewritten as:

$$\log(y/L) = -be^{-kt} \quad (8)$$

Rewritten as,

$$Y = \log[\log(L/y)] = \log b - kt \quad (9)$$

When $\log[\log(L/y)]$ is plotted against time '*t*', we get a straight line. And when '*Y*' is regressed on '*t*', the constant term is '*log b*' and slope term is '*k*', the slope term is taken to be intrinsically positive.

The straight line obtained by this transformation slopes down to the right. Once '*b*' and '*k*' are obtained from the regression, they can be substituted in the formula. Future values of '*t*' may then be substituted into the formula to obtain forecasts of the variable '*y*'. This growth curve is used to predict the state of technology for which there is a limit, and when the growth in the initial stages is comparatively faster than that of the Pearl curve. Pearl and Gompertz curves have been successfully used for predicting literacy in India (Katiyar et al., 2010).

III. Data Table And Parameter Estimates

Above all mathematical models contain certain parameters that have to be estimated from past data. Depending upon the characteristics of models and the availability of data, different techniques for parameter estimation have been suggested by researchers such as (i) ordinary least square (OLS) (ii) non-linear least square (NLS) with the use of SYSTAT statistical tools (SYSTAT, 1994). Both cases are analysed including engineering students' enrollments in USA and U.G., P.G., and Ph.D. students' enrollments in India. Of the four models, viz. Linear, Exponential, Pearl and Gompertz applied on the past data, results of any one model was found to be more suitable as this model gave a better fit over the past data. In our opinion NLS method yields better results. Results of parameter estimates and projections are given in further cases.

A. Case 1: Engineering Enrollment (USA)

TABLE I
ENGINEERING ENROLLMENT (USA)

Year	U.G.	P.G.	Ph.D.
1999	364858	70752	38055
2000	373073	75368	39467
2001	389993	78947	41446
2002	397878	89442	47263
2003	408766	91665	52604
2004	409778	87914	56001
2005	397437	83293	57077

Source: Annual Reports (1995-2006), University Grant Commission, New Delhi

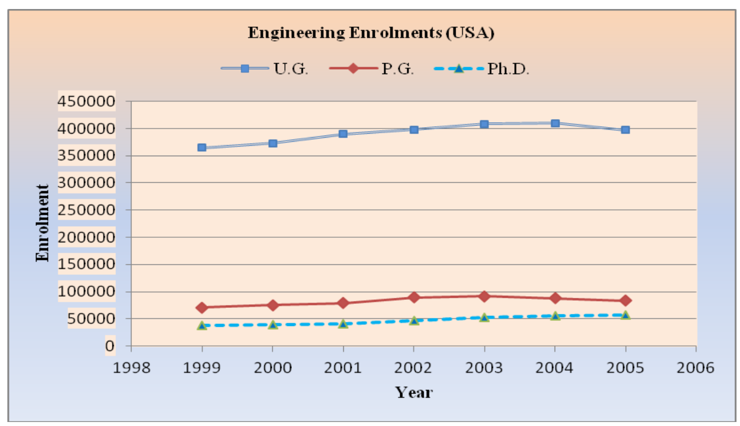

Fig. 1 Engineering Enrollment in USA

TABLE II
PARAMETER ESTIMATES (U.G.)

	Linear	Exponential	Pearl	Gompertz
A	364551.831	365669	-23383.972	
B	6782.863	0.017	8769.127	-23383.972
K				11692.136
L			391683.286	391683.286
MSE	.537599E+12	.537970E+12	.357970E+12	.357970E+12

TABLE III
PARAMETER ESTIMATES (P.G.)

	Linear	Exponential	Pearl	Gompertz
A	71706.852	72647.535	-5155.082	
B	2694.037	0.031	1841.237	-5535.666
K				2029.881
L			82483.000	82483.000
MSE	.239137E+11	.239094E+11	.158747E+11	.158747E+11

TABLE IV
PARAMETER ESTIMATES (PH.D.)

	Linear	Exponential	Pearl	Gompertz
A	32945.856	34568.440	-2983.486	
B	3617.572	0.076	1147.633	-3176.316
K				1197.011
L			47416.142	47416.142
MSE	.805223E+10	.805187E+10	.524601E+10	.524601E+10

TABLE V
PROJECTED VALUES FOR THE YEARS 2007-2015

Year	U.G.	P.G.	Ph.D.
2007	426270	96265	68591
2009	441046	102479	79872
2011	456334	109094	93009
2013	472152	116137	108307
2015	488518	123633	126121

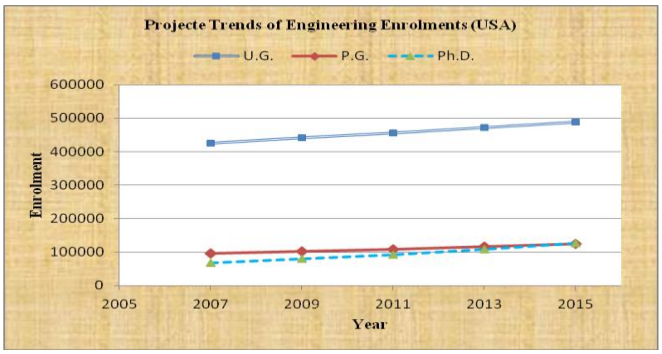

Fig. 2 Projected Trends for (2007-2015) of Engineering Enrollments in USA

B. Case 2: Total Enrollment (India)

TABLE VI
TOTAL ENROLLMENT (INDIA)

Year	U.G.	P.G.	Doctorate
1996	5890755	565465	53193
1997	6124125	597358	54740
1998	6491540	642547	55905
1999	7183557	733410	57159
2000	7183557	733410	57159
2001	7488736	775303	57411
2002	7961969	844083	60698
2003	8227417	846556	62213
2004	8867378	913732	65491
2005	9315808	986518	68838

Source: Annual Reports (1995-2006), University Grant Commission, New Delhi

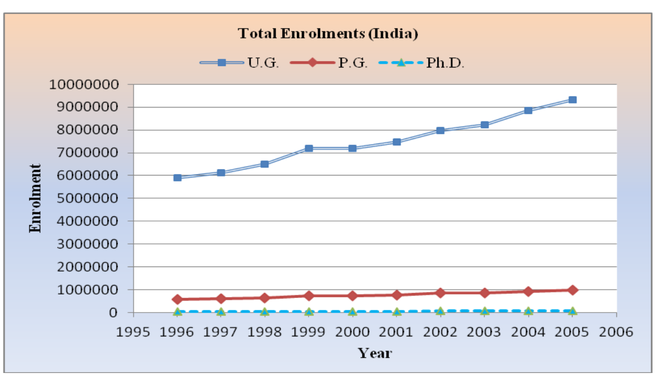

Fig. 3 Total Enrollment in India

TABLE VII
PARAMETER ESTIMATES (U.G.)

	Linear	Exponential	Pearl	Gompertz
A	5428549.016	5619599.003	-0.965	
B	371806.381	0.050	-0.001	5.505
K				0.010
L			200481.885	.137366E+10
MSE	.284967E+15	.284990E+15	.189981E+15	.189991E+15

TABLE VIII
PARAMETER ESTIMATES (P.G.)

	Linear	Exponential	Pearl	Gompertz
A	517236.444	545782.268	-49815.428	
B	44836.680	0.059	16605.276	-48755.523
K				16251.974
L			763838.201	763838.201
MSE	.300017E+13	.299995E+13	.194483E+13	.194483E+13

TABLE IX
PARAMETER ESTIMATES (PH.D.)

	Linear	Exponential	Pearl	Gompertz
A	50664.997	50981.159	-3187.030	
B	1566.491	0.027	1657.408	-3584.905
K				1536.531
L			59280.700	59280.700
MSE	.176722E+11	.176739E+11	.117140E+11	.117140E+11

TABLE X
PROJECTED VALUES FOR THE YEARS 2007-15

Year	U.G.	P.G.	Ph.D.
2007	10161800	1102193	69463
2009	11154700	1239170	72596
2011	12222900	1393169	75729
2013	13370100	1566307	78862
2015	14600100	1760962	81995

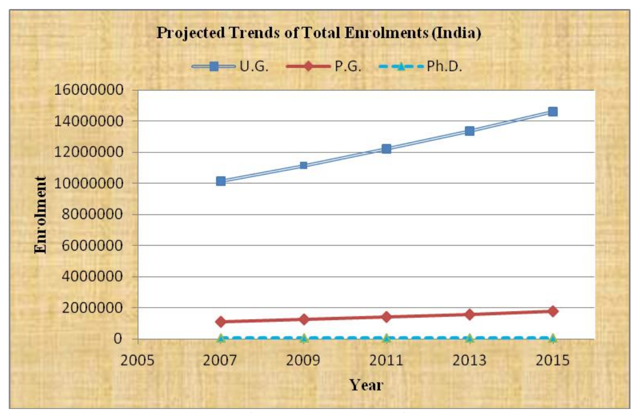

Fig. 4 Projected Trends for (2007-2015) of Total Enrollments in India

IV. RESULTS AND DISCUSSION

In the present study applications of the Linear, Exponential, Pearl and Gompertz models have been made to analyse the cases of engineering enrollments in USA and discipline-wise enrollments in India. For both cases, we have estimated the model parameters using quasi-Newton method and simplex method, depending on the convergence of the model. Parameter estimates and mean square error (MSE) are listed in Table II to Table IV for the case-1 and from Table VII to Table IX for the case-2. Using these parameter values, future projections have been calculated and are listed in Table V and

Table X for the first and second cases respectively. A brief description about model applicability and results arrived at each case is given below:

A. Case 1: Engineering Enrollment (USA)

U.G. Enrollment:

In the case of engineering enrollments in USA has three different disciplines are under graduate, post graduate and doctorate. Firstly, four given models were applied on the past data of U.G. enrollments. After applying these models, we got some parameters and in the Pearl model it was found that the value of parameter *L*=upper limit (Table II) (391683.286<409778) is less than the actual maximum value (Table 1). So, this model is not fit for this case and further remaining three models are checked for this case. First, minimum (least) mean square error (MSE) values are selected. From Table II, least MSE value was found for Gompertz model. Then this model was analysed for further fit but we did not get the best-fit projection. In this case again we selected the least MSE from the remaining linear and exponential models and we got that the linear model is the most reliable for the best projection as the actual values of U.G. enrollments is very nearest from the estimated line. The projected values for the years 2007-2015 are given in Table V.

P.G. Enrollment:

Similar all four models were applied on the past data of P.G. enrollments and found that the value of *L*=upper limit (Table III) (82483<91665) is less than the maximum value for post graduate past data (Table I). So this model is not applicable for this case and again remaining three models were analysed and found that least MSE value is for exponential model. Thus, exponential model was the most reliable for the best fit.

Ph.D. Enrollment:

In the case of doctorate enrollments, linear, exponential, Pearl and Gompertz models are also applied. From the Table IV, it is found that the value of *L*=upper limit (47416.142<57077) is less than the maximum value of the Ph.D. past data (Table I). So, Pearl and Gompertz models again do not fit for this case also. Finally, on the basis of minimum value of MSE, it was found that the exponential model is the best fit for the Ph.D. enrollment case.

B. Case 2: Total Enrollments (India)

U.G. Enrollment:

In this case of total enrollments in India is having three different disciplines (U.G., P.G., and Ph.D.). Firstly, four given models were applied on the past data of U.G. enrollments. After applying these models, we got some parameters and in the Pearl model it was found that the value of parameter *L*=upper limit (Table VII) (200481.885<9315808) is very less than the actual past data of U.G. enrollment (Table VI). So, this model is not fit for this case and further remaining three models are checked for this case. First, minimum (least) mean square error (MSE) values are selected. From Table VII, least MSE value was found for Gompertz model. Then this model was analysed for further fit but we did not get the best-fit projection. In this case again we selected the least MSE from the remaining linear and exponential models and we got that the linear model is the most reliable model for the best projection as the actual values of U.G. enrollments is very nearest from the estimated line. The projected values for the years 2007-2015 are given in Table X.

P.G. Enrollment:

Similar all four models were applied on the past data of P.G. enrollments and found that the value of *L*=upper limit (Table VIII) (763838.201<986518) is less than the maximum value for post graduate past data (Table VI). So this model is not applicable for this case and again remaining three models were analysed and found that least MSE value is for exponential model. Thus, exponential model was the most reliable for the best fit.

Ph.D. Enrollment:

In the case of doctorate enrollments, linear, exponential, Pearl and Gompertz models are also applied. From the Table IX, it is found that the value of *L*=upper limit (59280.700<68838) is less than the maximum value of the Ph.D. past data (Table VI). So, Pearl and Gompertz models again do not fit for this case also. Finally, on the basis of minimum value of MSE, it was found that the linear model is the best fit for the Ph.D. enrollment case.

V. Conclusion

This paper analyses engineering students' enrollments in USA and students' enrollments of different discipline (U.G., P.G., and Ph.D.) in India with the help of various mathematical models such as linear, exponential, Pearl and Gompertz for the purpose of manpower forecasting and policy studies. In the present study applications of the all four models have been used to analyse the mathematical modelling of students' enrollments in different disciplines of education in USA and India. From the results, it was found that under graduate and post graduate students' enrollment trends are not much increasing compare to Ph.D. enrollments in USA. It means, doctorate degree is going to be in demand in USA. In the case of India, under graduate degree is in more demand compare to post graduate and doctorate. However, the trends of P.G. and Ph.D. are increasing too. As technical education plays a significant role to lead nation towards development. It has been found that NLS method yields better results. On the basis of analyses of models fit and forecasts, parameters were identified. Further, it is suggested that these models are the best fit for estimation of future projection. Also it is concerned that the performance of the Pearl and Gompertz models in this study is very poor. Resulting forecasts may be useful for planning strategies to meet the future national as well as international requirements in the field of education.

References

[1] Annual Reports, *University Grant Commission*. New Delhi, 1995–2006.

[2] B. Gompertz, *Philosophical Transactions Royal Society*. London, 1825, vol. 115, pp. 513–585.

[3] L.P. Rai, and N. Kumar, "S&T Education in India: Prospects and Challenges," *Scientometrics*, vol. 61, no. 2, pp. 157–169, 2004.

[4] L.P. Rai, and N. Kumar, "Scientific and Technical Manpower Production in India: The Changing Scenario," *Manpower Journal*, vol. XXXXI, no. 1, pp. 45–54, 2006.
[5] L.P. Rai, and R. Katiyar, "Emerging Trends in Higher Education in India," *CAMIST Conference 2010*, NIT Rourkela, Orissa, pp. 936–943.
[6] R. Katiyar, "Mathematical Modelling and Forecasting: Some Applications in Manpower Development," *M. Tech. Thesis*, School of Future Studies and Planning, DAVV Indore, M.P., India. 2007.
[7] R. Katiyar, L.P. Rai, and M.K..Barua, "Growth Pattern of Science and Engineering Education in India," in *ISDASEAST Conference 2010*, Department of Statistics, University of Jammu, Jammu, pp. 97.
[8] R. Pearl, "The Growth of Populations," *Quarterly Review of Biology*, vol. 2, pp. 532–548, 1972.
[9] SYSTAT, SYSTAT Inc., *IL 60201 (312) 864–5670*. Evanston, 1800 Sherman Avenue, 1994.

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