

# A Study on Mortgage Default Risk Factors by Applying Heckman Two-stage Regression Model

Po-Sheng Ko<sup>1</sup>, Cheng-Chung Wu<sup>2</sup>, Shu-Man Tsai<sup>3</sup>, and Min-Chih Lu<sup>4</sup>

**Abstract**— The real estate mortgage loans have been a very important business in consumer loans of commercial banks. In practice, there are many factors affecting the borrower's risk behavior, therefore, the decision-making of possible default of the borrower is an uncertain concept to the financial institutions. Through data analysis, this study identified the factors that can explain the mortgage default, and compared the findings with previous studies in Taiwan, in order to propose the major risk factors affecting the default behaviors of mortgage loan borrowers. These factors are risk factors that should be considered by financial institutions in mortgage business. The findings can provide a reference to loan-giving decisions.

**Keywords**---Heckman two-stage regression model, default risk, Two-stage Regression Model.

## I. INTRODUCTION

REGARDING the loan default risk of financial institutions, the major issue is the estimation of the risk occurrence probability. Therefore, financial institutions' decision of possible default is an uncertainty in doing mortgage business. However, when financial institutions undertaking the mortgage, if the market interest rate is below the nominal interest rate, as the mortgage balance is less than the present value of the mortgage, the borrower will repay the loan by refinancing to enjoy lower interest rate cost. When the house present is below the mortgage balance, the borrower will have negative equity to default to increase the wealth. This phenomenon is the principle of European options, that is, the European put option is out of money, the default option value is equal to the deduction of the mortgage loan market value by house price. If in the state of out-of-money option, the real value of the mortgage loan is the deduction of mortgage loan market value by the values of prepayment and default options. Deng, Quigley and Van Order (2000) pointed out that call and put options of the mortgage loan have a great impact on

prepayment and default. They further argued that the borrower prepayment and default decisions are mutually affecting, that is, the prepayment decision is affected by default, and vice versa.

## II. LITERATURE REVIEW

Many subsequent studies applied the model in real estate mortgage loan market. For example, Dunn and McConnell (1981), on the assumption of optimal prepayment policy, combined Poisson prepayment model and interest rate model to obtain the GNMA transfer-securities prices by implicit finite differentiation method. The features of the strategy include:

(1) It is assumed that the mortgage borrowers are homogeneous, and each borrower has the same interest expectations and risk preferences and thus will have the same behaviors.

(2) The market is perfect without trading and refinancing costs.

On the same assumption, Kau et al.(1992, 1995) considered both default and prepayment options. In the strategy, the borrower prepayment behavior is completely determined by whether the mortgage balance is lower than the present value of mortgage; the borrower default behavior is completely determined by whether the house market value is lower than the mortgage balance. The findings suggest that the exercising of each option is determined by the house price and interest rate.

Dawkins (2002) used the simulation method to estimate and compare the mortgage default behaviors by using the traditional banking single-variate evaluation equation (probit regression model) by considering traditional variables including: loan numbers, skin, default and not-to-default decisions, monthly income, the proportion of monthly repayment against income, loan balance, repayment grace period, previous loan conditions, loan term, loss of repayment failure, credit, loss of major credit account due to repayment failure, borrower credit survey. It was found that the method can get relatively better results in controlling credit, income, credit conditions and repayment grace period after modifying skin color and loan balance. The author underlined that all the traditional variables should be considered in modification of the method.

<sup>1</sup>Associate Professor, Department of Public Finance and Taxation, National Kaohsiung University of Applied Sciences

<sup>2</sup>Assistant Professor, Department of Administration, Cheng Shiu University

<sup>3</sup>Master Student, Department of Public Finance and Taxation, National Kaohsiung University of Applied Sciences

<sup>4</sup>Master Student, Department of Industrial Engineering Management, National Kaohsiung University of Applied Sciences

### III. RESEARCH METHOD

#### *Research model- Heckman two-stage regression*

Due to limitation of data, previous studies on default and prepayment at home and abroad mainly discussed the data types and had no conclusion on the existence of default and prepayment. Moreover, the impact of self-selection on default and prepayment has not been discussed in depth. Therefore, this study used the two-stage program proposed by Heckman (1979) to discuss the endogenous problem of default and prepayment choice and considered other models of incomplete limitations. At Stage I, this study conducted the logistic regression analysis of model (1) and model (2), and calculated the Inverse Mill's ratio. Heckman (1979) proposed the two-stage program to use the Inverse Mill's ratio to estimate the possible self-selection problem. The sample selection bias is one of the key points of study of limited dependent variable in econometrics. It means that the samples cannot fully represent the population of concern to the researcher. As the research data type is non-random, to avoid the self-selection bias problem (Heckman, 1979), therefore, a control variable, inverse Mill's ratio, should be added in the model as the inverse Mill's ratio can reduce the occurrence of the bias. Inverse Mill's ratio can be deduced by the selection function Probit model. First, Probit regression model can be used to estimate the coefficient of the model default behavior and calculate the inverse Mill's ratio. Then, the inverse Mill's ratio can be added in the model as the control variable to modify the self-selection bias. Heckman proposed the two-stage modification method, also known as the Type II Tobit model. First, at Stage I, the Probit model is estimated and the inverse Mill's ratio is calculated. Next, the ratio is used as an additional independent variable in the OLS model for control and estimation. The equation for the calculation of inverse Mill's ratio is:  $[\phi(Z)/\Phi(Z)]$ , where,  $\phi$  represents the standard normal probability density function,  $\Phi$  represents the standard normal cumulative distribution function.

### IV. EMPIRICAL RESULTS

Most of literature on the problem of sample selection bias in Taiwan focused on consumer financial fields, for example, the studies by Liu (2004), Chen(2005). Few articles discussed the loan financial fields. Therefore, this study discussed the financial crisis early warning model that rejects the inference technology, that is, previously, the samples collected for the construction of the traditional financial crisis early warning models are samples of applicants that have been approved by the bank. As the credit quality of loan applicants varies, the financial crisis early warning model established by using the samples of approved applicants is not reliable as the samples are not representative. If such a model is used as the basis for the review of the second applicant, it can easily result in wrong decisions. If the samples of approved applicants are used to estimate the model parameters, it may result in bias of model parameter estimation.

The two-stage sample selection model proposed by Heckman(1979) regards the default or not to default and default stages regarded two stages to establish two models. In addition, Boyes et al. (1992), Greene (1986) applied the method in literature study. The articles suggested that there is a significant sample selection bias in loan decision making. Moreover, the mixture model applied by Feelders (1999) also belongs to the category. Feelders found that the original judgment standards will change if the rejection samples are added. Therefore, this study analyzed according to default and prepayment risk as follows:

#### 1. Default risk Heckman sample selection model

TABLE I  
HECKMAN SAMPLE SELECTION MODEL AND DEFAULT MODEL  
PARAMETER ESTIMATIONS

| Stage I (default or not)<br>N=2993 | Expected symbol | Estimation coefficient | Standard deviation | P-value |
|------------------------------------|-----------------|------------------------|--------------------|---------|
| Parameter                          | Negative        | -0.0004494             | 0.0007277          | 0.537   |
| Loan term                          | Negative        | -2.37e-08              | 1.30e-08           | 0.070   |
| Initial loan sum                   | Uncertain       |                        |                    |         |
| Balance of loan %                  | Positive        | 0.9786157              | 0.1203707          | 0.000** |
| Financial condition                | Negative        | -6.25e-07              | 5.86e-07           | 0.286   |
| Payment grace period               | Negative        | -0.0020096             | 0.0027387          | 0.463   |
| The loan percentage                | Positive        | 0.2752069              | 0.2105457          | 0.191   |
| Age                                | Negative        | -0.0015354             | 0.0036793          | 0.676   |
| Gender                             | Uncertain       | 0.0584826              | 0.0697816          | 0.402   |
| Constant                           |                 | -2.055117              | 0.2984717          | 0.000** |
| Stage II (default times)<br>N=245  |                 |                        |                    |         |
| Balance of loan %                  | Positive        | -9.213604              | 4.218857           | 0.029** |
| Constant                           |                 | 30.86787               | 11.77711           | 0.009** |
| Mill's lambda                      |                 | -11.19513              | 4.86545            | 0.021** |

### V. CONCLUSION

In general, financial institutions are concerned about whether the borrower is willing to and capable of repaying the loan as stipulated in the contract--repayment in time. Therefore, it needs to evaluate the borrower's loyalty to repayment, that is, regardless of the financial conditions of the borrower, the borrower should be willing to pay the debts as stipulated in the contract. This can be inquired from the previous individual credit records and transaction data with the financial institutions. Moreover, the willingness of the borrower to repay the loan can roughly be seen from the age and purpose of purchasing the house as older borrowers or borrowers buying the houses for living purpose have stronger willingness to repay the loans and thus their default risk will be lower.

### REFERENCES

- [1] Black, F. and M. Scholes. (1972). The Pricing of Options and Corporate Liabilities, *Journal of Political Economy*, 81, 637-659.  
<http://dx.doi.org/10.1086/260062>
- [2] Dawkins, M. C. (2002). Simultaneity bias in mortgage lending: A test of simultaneous equations models on bank-specific data. *Journal of Banking & Finance*, 26, 1593-1613.  
[http://dx.doi.org/10.1016/S0378-4266\(01\)00174-1](http://dx.doi.org/10.1016/S0378-4266(01)00174-1)
- [3] Deng, Y. H., Quigley, J. M., and Van Order, R. (2000). Mortgage terminations, heterogeneity and the exercise of mortgage options. *Econometrica*, 68(2), 275-307.  
<http://dx.doi.org/10.1111/1468-0262.00110>
- [4] Dunn, K. and J. McConnell. (1981). Valuation of GNMA Mortgage-Backed Securities, *Journal of Finance*, 36, 599-616.  
<http://dx.doi.org/10.1111/j.1540-6261.1981.tb00647.x>

- [5] Foster, C., and R. Van Order, (1984). An Option-based Model of Mortgage Default, *Housing Finance Review*, 3(4), 351-372.
- [6] Green, J. and Shoven, J. B. (1986). The Effects of Interest Rates on Mortgage Prepayments, *Journal of Money, Credit and Banking*, 18(1), 41-59.  
<http://dx.doi.org/10.2307/1992319>
- [7] Heckman, J. J. (1979). Sample selection bias as a specification error. *Econometrica*, 47(1), 153-162.  
<http://dx.doi.org/10.2307/1912352>
- [8] Kau, J. B., and Keenan, D.C., and Muller, W.J., and Epperson, J.F., (1992). A Generalized Valuation Model for Fixed-Rate Residential Mortgage, *Journal of Money, Credit, and Banking*, Vol. 24, No. 3, 279-298.  
<http://dx.doi.org/10.2307/1992718>
- [9] Kau, J. B., and Keenan, D.C., and Muller, W.J., and Epperson, J.F., (1995). The Valuation at Origination of Fixed-Rate Mortgages with Default and Prepayment, *Journal of Real Estate Finance and Economics* 11, No.1, 5-36.  
<http://dx.doi.org/10.1007/BF01097934>