Sudden Changes in Global Volatility Index

Sang Hoon, Kang, Suyeol, Ryu, and Seong-Min, Yoon

Abstract—This study considers the impact of sudden changes on volatility index. We employs an iterated cumulative sums of squares algorithm to identify the time points at which sudden changes in volatility occurred, and the results are incorporated into the GARCH framework with and without sudden change variables. The degree of persistence of volatility was reduced by incorporating these sudden changes into the volatility model.

Keywords—Sudden changes, ICSS algorithm, GARCH model, Volatility persistence.

I. Introduction

THE volatility of stock returns is affected substantially by infrequent sudden changes or regime shifts, corresponding to domestic and global economic events. However, the GARCH approach is incapable of incorporating sudden changes and is also inappropriate for examinations of volatility persistence [1-2]. In order to overcome this problem, Inclán and Tiao [3] designed the iterated cumulative sums of squares (ICSS) algorithms, which identify time points at which sudden changes in variances occur.

Using the ICSS algorithms, the impact of sudden changes on volatility has been extensively documented and modeled by popular GARCH class models [4-6]. The results of these studies unanimously support the notion that the incorporation of sudden changes into GARCH-type of models results in a significant reduction in the persistence of volatility in international stock markets.

This study re-examines the impacts of sudden changes on volatility persistence in Global volatility index, namely VIX. The principal objectives of this study are twofold: First, this study detects the sudden changes using the ICSS algorithm. Second, we examines whether the inclusion of sudden changes in the GARCH model reduces the coefficients of volatility asymmetry and persistence or not.

The reminder of the paper is organized as follows. Section 2 briefly presents the methodology for the ICSS algorithm and for the GARCH model. Section 3 describes the characteristics of the sample data. Section 4 incorporates the sudden changes in volatility. The final section, Section 5, provides some concluding remarks.

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II. Methodology

In accordance with the work of Inclán and Tiao [3], this study identifies sudden changes in volatility with the ICSS algorithm, and then estimates the univariate GARCH$(1,1)$ mode with and without sudden change dummies.

A. Detecting points of sudden change in variance

The ICSS algorithm was utilized to identify discrete sub-periods of the changing volatility of stock returns. It assumes that the variance of a time series is stationary over an initial period of time, until a sudden change occurs as the result of a sequence of financial events; the variance then reverts to stationary until another market shock occurs. This process is repeated over time, generating a time series of observations with an unknown number of changes in the variance.

Let $\{\varepsilon_t\}$ denote an independent time series with a zero mean and an unconditional variance, σ_t^2. The variance in each interval is given by σ_j^2, $j = 0,1,\ldots,N_T$, where N_T is the total number of variance changes in T observations and $1 < K_1 < K_2 < \ldots < K_{N_T} < T$ are the set of change points. The variance over the N_T intervals is defined as follows:

$$\sigma_t^2 = \begin{cases} \sigma_0^2, & 1 < t < K_1 \\ \sigma_1^2, & K_1 < t < K_2 \\ \vdots & \\ \sigma_{N_T}^2, & K_{N_T} < t < T \end{cases} \qquad (1)$$

A cumulative sum of squares is utilized to determine the number of changes in variance and the time point at which each variance shift occurs. The cumulative sum of squares from the first observation to the k^{th} point in time is expressed as follows:

$$C_k = \sum_{t=1}^{k} \varepsilon_t^2 , \quad \text{where } k = 1,\ldots,T . \qquad (2)$$

Define the statistic D_k as follows:

$$D_k = \left(\frac{C_k}{C_T}\right) - \frac{k}{T}, \quad \text{where } D_0 = D_T = 0 , \qquad (3)$$

in which C_T is the sum of the squared residuals from the whole sample period. Note that if no changes in variance occur, the D_k statistic will oscillate around zero (if D_k is plotted against k, it will resemble a horizontal line). However, if one or more changes in variance occur, then the statistic values drift up or down from zero. In this context, significant changes in variance are detected using the critical values obtained from the distribution of D_k under the null hypothesis of constant variance. If the maximum absolute value of D_k is greater than the critical value, the null hypothesis of homogeneity can be rejected. Define k^* as the value at which $\max_k |D_k|$ is reached, and if $\max_k \sqrt{(T/2)}\ |D_k|$ exceeds the critical value, then k^* will be used as the time point at which a variance change in the series occurs. The term $\sqrt{(T/2)}$ is required for the standardization of the distribution.

In accordance with the study of Inclán and Tiao (1994), the critical value of 1.358 is the 95th percentile of the asymptotic distribution of $\max_k \sqrt{(T/2)}\ |D_k|$. Therefore, the upper and lower boundaries can be established at ± 1.358 in the D_k plot. A change point in variance is identified if it exceeds these boundaries. However, if the series harbors multiple change points, the D_k function alone will not be sufficiently powerful to detect the change points at different intervals. In this regard, Inclán and Tiao [3] modified an algorithm that employs the D_k function to search systematically for change points at different points in the series. The algorithm works by evaluating the D_k function over different time periods, and those different periods are determined by breakpoints, which are themselves identified by the D_k plot.

B. GARCH(1,1) model

Following the seminal work of Engle [7] considers the return series y_t and the associated prediction error $\varepsilon_t = y_t - E_{t-1}[y_t]$, in which $E_{t-1}[\cdot]$ is the expectation of the conditional mean on the information set at time $t-1$. The GARCH $(1,1)$ model of Bollerslev [8] is as follows:

$$y_t = \mu + \varepsilon_t,\ \varepsilon_t = z_t\sqrt{h_t},\ z_t \sim N(0,1), \quad (4)$$

$$h_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}, \quad (5)$$

where $\omega > 0$, $\alpha \geq 0$, $\beta \geq 0$, which ensures that the conditional variance (h_t) is positive, and $(\alpha + \beta) < 1$ are introduced for covariance stationarity. In the GARCH model, the sum of α and β quantifies the persistence of shocks to conditional variance. A common empirical finding is that the sum of α and β is quite close to one, thereby implying that shocks are infinitely persistent, corresponding to an integrated GARCH (IGARCH) process.

C. Multiple sudden changes with GARCH model

In an effort to asses the impact of sudden changes on volatility, sudden changes should be incorporated into the standard GARCH model. Following the study of Aggarwal Inclán and Leal [4], we modify above GARCH (1,1) and GJR-GARCH (1,1) models in Equations (5)-(6) with multiple sudden changes that were identified via the ICSS algorithms, as follows:

$$h_t = \omega + d_1 D_1 + \cdots + d_n D_n + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}, \quad (7)$$

in which $D_1,\ldots,D_n$ are dummy variables that take a value of one from each point of sudden change of variance onwards, and take a value of zero elsewhere.

III. Data

This study considers CBOE Volatility index(VIX) provided by the database of the DataStream. The data sets consist of the weekly Friday closing prices spanning from January 3, 2003 to September 28, 2012 (509 observations). Figure 1 shows the dynamics of VIX.

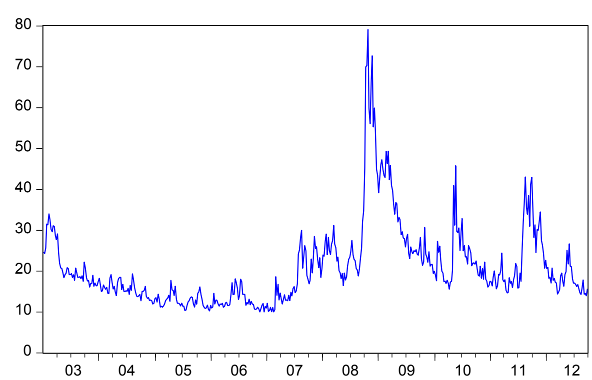

Fig. 1 The dynamics of VIX

The price series are converted into the logarithmic percentage return series for all sample indices, i.e. $r_t = 100 \times \ln(P_t / P_{t-1})$ for $t = 1,2,\ldots,T$, where r_t is the returns for each index at time t, P_t is the current price, and P_{t-1} is the price from the previous week. Table 1 provides the descriptive statistics and the results of the unit root test for both sample returns. As is shown in Panel A of Table 1, the means of both return series are quite small, and the corresponding standard deviations of returns are substantially higher. The distribution of returns is not normally distributed, as is indicated by the skewness, kurtosis, and Jarque-Bera tests.

TABLE I
DESCRIPTIVE STATISTICS AND UNIT ROOT TESTS

	VIX
Panel A: Descriptive Statistics	
Mean	-0.088
Std. Dev.	12.61
Maximum	70.71
Minimum	-43.36
Skewness	0.672
Kurtosis	6.031
Jarque–Bera	233.30 [0.000]
Panel B: Unit root tests	
ADF	-28.22***
PP	-28.99***
KPSS	0.056

Notes: *** indicates a rejection of the null hypothesis at the 1% significance level.

Additionally, Panel B of Table 1 provides the results of three types of unit root test for each of the sample returns: the augmented Dickey-Fuller (ADF), Phillips-Peron (PP) and

Kwiatkowski, Phillips, Schmidt, and Shin (KPSS). The null hypothesis of the ADF and PP tests is that a time series contains a unit root, whereas the KPSS test has the null hypothesis of a stationary process. As is shown in Panel B, large negative values for the ADF and PP test statistics reject the null hypothesis of a unit root, whereas the KPSS test statistic does not reject the null hypothesis of stationarity at a significance level of 1%. Thus, both return series are a stationary process.

IV. Empirical Results

A. Sudden changes in variances

The ICSS algorithm calculates the standard deviations between the change points to determine the number of sudden changes. Fig. 2 illustrates the returns of the Shanghai and Shenzhen series with the points of sudden change and ±3 standard deviations.

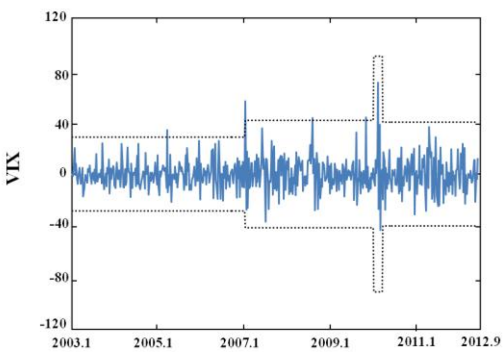

Fig. 2 Sudden changes in VIX

TABLE II
Sudden Change Point estimated by ICSS algorithm

No of change points	Time period
1	3 January 2003–23 February 2007
	2 March 2007– 28 September 2012

Note: Time periods were detected by the ICSS algorithm.

Table 2 indicates the time periods of sudden changes in volatility as identified by the ICSS algorithm. Looking at Figure 2 and Table 2, the time points of sudden change in volatility are correlated to global economic events. For example, sudden change was related the global financial crisis in 2007.

B. GARCH estimation with and without sudden changes

The next step is to incorporate these sudden changes in the GARCH model and examine the impact of sudden changes in volatility. Table 3 reports the estimation results from the GARCH (1,1) model with and without sudden change dummy variables.

In the GARCH (1,1) model of Table 3, the model without the dummy variables evidences highly significant α and β, and the sums of the parameters (0.742), which is reflective of volatility persistence, i.e. shocks have a permanent impact on the variance of returns. However, the inclusion of dummy variables dramatically reduces the sum of the parameters (0.324). This evidence is consistent with the studies of Aggarwal Inclán and Leal [4] and others, whom have argued that the standard GARCH model overestimates volatility persistence when ignoring sudden changes in conditional variance.

TABLEIII
The estimation of GARCH(1,1) model

	α	β	$\alpha+\beta$
With dummies	0.140 (0.602)***	0.602 (0.117)***	0.742
Without dummies	0.005 (0.024)	0.319 (0.296)	0.324

Notes: *** indicates significance at the 1% level.

V. Conclusion

In this study, we have investigated sudden changes of volatility and examined the persistence of VIX. In an effort to assess the impact of sudden changes in volatility, we identify the time points at which sudden changes in volatility occur, and then incorporate this information into the GARCH model. Using the ICSS algorithm, the identification of sudden changes is largely associated with domestic and global events. When these sudden changes are incorporated into the GARCH and model, the evidences of persistence has been vanished in the volatility.

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